Is it that easy?*

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Abstract

In this paper we provide one single aspect of the financial crisis. We show in a very simple model the dramatic downgrade of a simplified CDO^2, if the default probability of the underlying credit varies slightly.

Keywords: ...

JEL Classification: ...

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1 Example

Suppose in January 2002 there are a thousand ($n = 1000$) Jeffs sitting in California, Florida, Arkansas or where ever. They do not have much money, but they have a good Job and they dream of their own houses. Usually it would be hard to get credit, because each has a default probability of more than 5 percent within 10 years. But their mutual savings banks surprisingly offer every Jeff a credit of 1 house unit ($I = 1$) with very low redemption rates and they do not ask for any securities (recovery rate is zero) and claim, that Jeff can repay the credit not only with his labor income, but also with the future price of his house, which is assumed to rise during the next years (see figure 3).

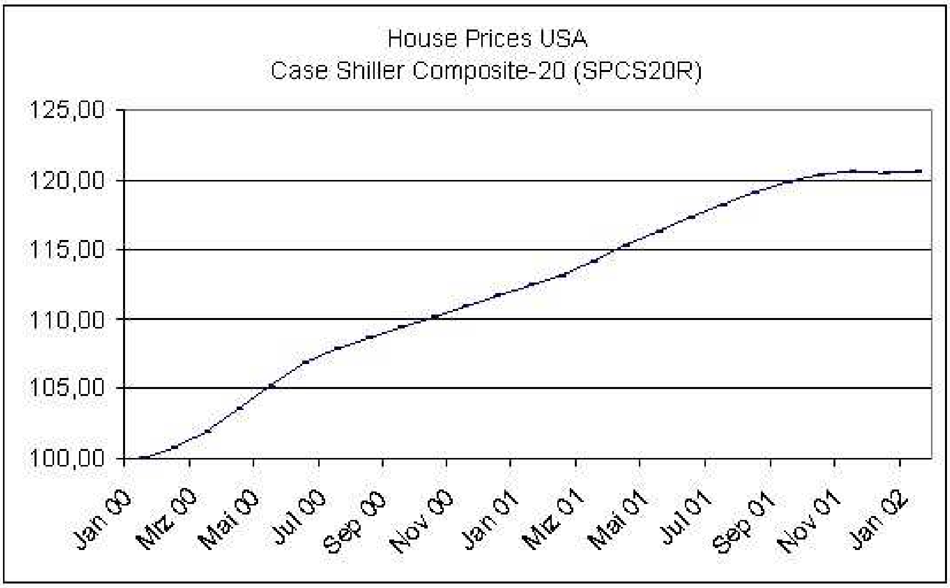

Figure 1: USA House Prices until January 2005

With this assumption Jeff's default probability is estimated at $p = 5$. Now there comes Fannie buying the 1000 credits and tying them into three packages. The first package, call it equity trache (eq) takes the first 6.5% of the losses. The second, call it mezzazine trache (mz) takes the next 6.5% of the losses and the remainig package we call senior trache (sen). For simplicity, we assume, that all Jeffs have not anything to do with each other (all correlations between different Jeffs are zero). Now we can calculate the default probabilities of the three packages. Generally the default probability is given by

$$P_d(X) = \sum_i^n \binom{n}{i} p^i (1-p)^{n-i} \tag{1}$$

and we obtain

$$\begin{aligned} P_d(eq) &= \sum_1^{1000} \binom{n}{i} p^i (1-p)^{n-i} \approx 98,5\% \\ P_d(mz) &= \sum_{66}^{1000} \binom{n}{i} p^i (1-p)^{n-i} \approx 1,5\% \\ P_d(sen) &= \sum_{130}^{1000} \binom{n}{i} p^i (1-p)^{n-i} < 0.01\% \end{aligned} \tag{2}$$

Of course there are not only Jeffs, but also Jims and Johns and they all run through the same procedure. Now Fannie has three equity, mezzazine and senior tranches (additionally, we assume that Jeff, Jim and John also do not have anything to do with each other) and she is knocking on Mr Fitch's door, asking him, how he would grade these packages, because he is known as a rating expert (his measure is the so called exceedance probabiility[1]). He looks up his table[2] and answers: "eq is really bad, I cannot grade it anyhow (NR), but mz is quite good, I grade it $A+$ and sen is marvellous, it gets AAA." (See also figure 2))

Rating	AAA	AA+	AA	AA-	A+	A	A-		
10y Def Rate	0.19	0.57	0.89	1.15	1.65	1.85	2.44		
Rating	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
10y Def Rate	3.13	3.74	7.26	10.18	13.53	18.46	22.84	27.67	34.98
Rating	CCC+	CCC	CC	C					
10y Def Rate	43.36	48.52	77.0	95.0					

[1]See Whetten und Adelson 2005

[2]See Coval, Jurek and Stafford (2008).

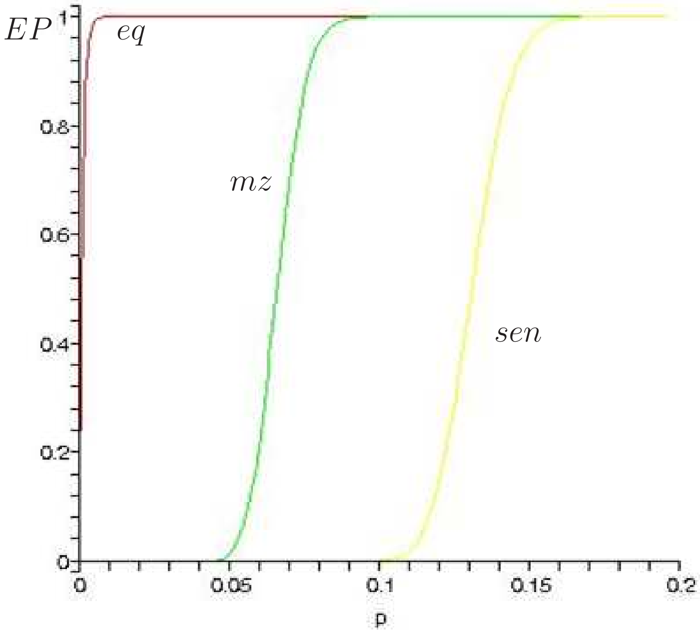

Figure 2: Exceedance Probabilities (EP) of the different tranches depending on the default probability p of one single credit.

Of course, there is not only Fannie but also Freddie and Indy and they are doing the same with Ben, Bill and Bob and Ted, Tim and Tom (again, all these guys have nothing in common). What comes next? There are two smart brothers called Lehman (do not confuse them with the bad german soccer goal keeper) looking for a good investment idea. They think, the *sen* is not very risky, so there is not much to earn, but the mz is not bad, and if we buy the three mezzazine tranches, we can square and repackage them and sell the resulting tranches further. No sooner said than done. The repackaging is done as follows. The first tranche eq^2 takes the first losses, the next one (mz^2) the second losses and the last sen^2 does not pay only if all three mz's default. Again we calculate the default probabilities

$$
\begin{aligned}
P_d(eq^2) &= \sum_1^3 \binom{3}{i} P_d(mz)^i (1-P_d(mz))^{n-i} \\
&= 3P_d(mz)(1-P_d(mz))^2 + 3P_d(mz)^2(1-P_d(mz)) + P_d(mz)^3 \quad \approx \quad 4,4\% \\
P_d(mz^2) &= \sum_2^3 \binom{3}{i} P_d(mz)^i (1-P_d(mz))^{n-i} \\
&= 3P_d(mz)^2(1-P_d(mz)) + P_d(mz)^3 \quad \approx \quad 0.07\% \\
P_d(sen^2) &= \sum_3^3 \binom{3}{i} P_d(mz)^i (1-P_d(mz))^{n-i} \\
&= P_d(mz)^3 \quad < \quad 0.01\%
\end{aligned}
\tag{3}
$$

and again Mr Fitch is asked for grading. He looks up his table and says:"$BBB-$ for eq^2, hmm not really good, but the other two packages are excellent, I assigning both mz^2 and sen^2 with AAA." And now? You know, we are living in a globalized world and we already spoke of a german goal keeper, who was playing some time in North-Rhine-Westfalia. Coincidentally in Düsseldorf, the capital of North-Rhine-Westfalia, a bank called ABS is located. Her asset managers have heard of this top graded mz^2 package with almost no default probability and after they heard, that they can buy it for a in their eyes low price, they made a big deal and bought an amount of mz^2 of about $\frac{1}{5}$ of their total assets. Is it not nice? Jeff's little cute house is financed by some guys in Düsseldorf. Now time goes by and Jeff is transfering every month his rate for his credit until summer 2005. But then nothing really bad happens. Only house prices raise not that much (see figure **??**), as assumed, which means that the default probabilities of Jeff, Jim and John, Ben and Bill, Ted, Tim and Tom where not really accurat. Actually, they are not 5%, but slightly higher with 7%. This should not change very much or?

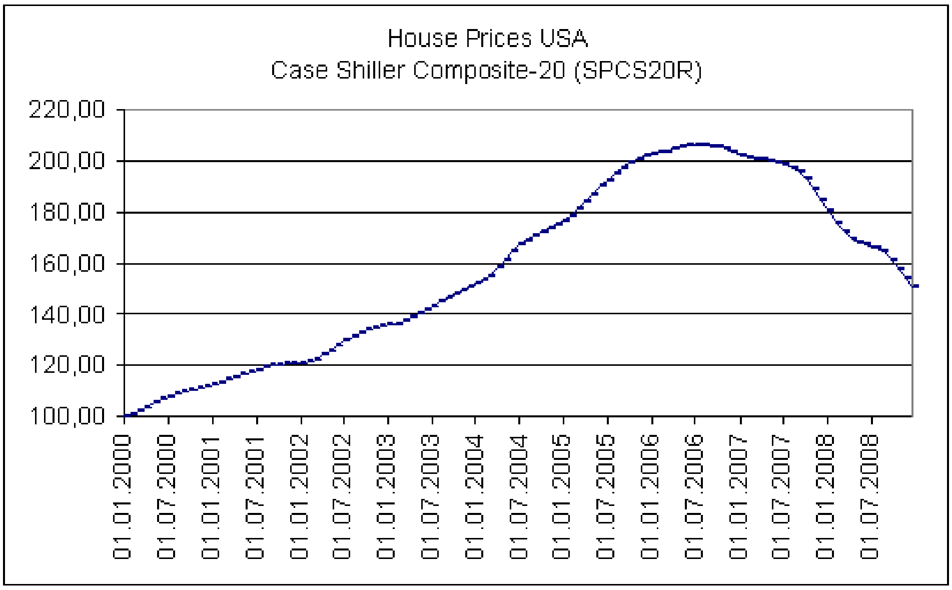

Figure 3: USA House Prices until December 2008

But astonishingly, if we go throug all the calculations again, we obtain a default probability of 79.3% for mz^2 and a look in Mr Fitch table grades it then only C! What does this mean. Due to accounting standards, the ABS bankers have to write down their mz^2 position almost totally, which means, they have not enough equity in relation to their total assets any more. But there are other banks, which are closely connected to ABS, so they also have to rebalance because of the dramatically raised default probability of ABS and of course, not only ABS has bought these very high graded derivatives of Jeff's credit. What is the end of story...?

The dramatic change in default probabilities and thus in rating is provided in the following table 2 and in figure **??** we show the high non linearity of the default probability of $P(mz_2)$ within the intervall $[0, 0.07]$. The non linearity is not very surprising, since the elements of the binomial series, from which we calculate the default probabilities are polynomials of the degree of the number of credits packed together ($n = 1000$).

	eq_1	mz	sen	eq^2	mz^2	sen^2
$p = 5\%$	NR $(> 99,9\%)$	$A+$ $(1,5\%)$	AAA $(< 0.01\%)$	$BBB-$ $(4,4\%)$	AAA (0.07%)	AAA $(< 0.01\%)$
$p = 7\%$	NR $(> 99,9\%)$	CC $(70,7\%)$	AAA $(< 0.01\%)$	NR $(97,5\%)$	C $(79,3\%)$	$CCC+$ $(35,4\%)$

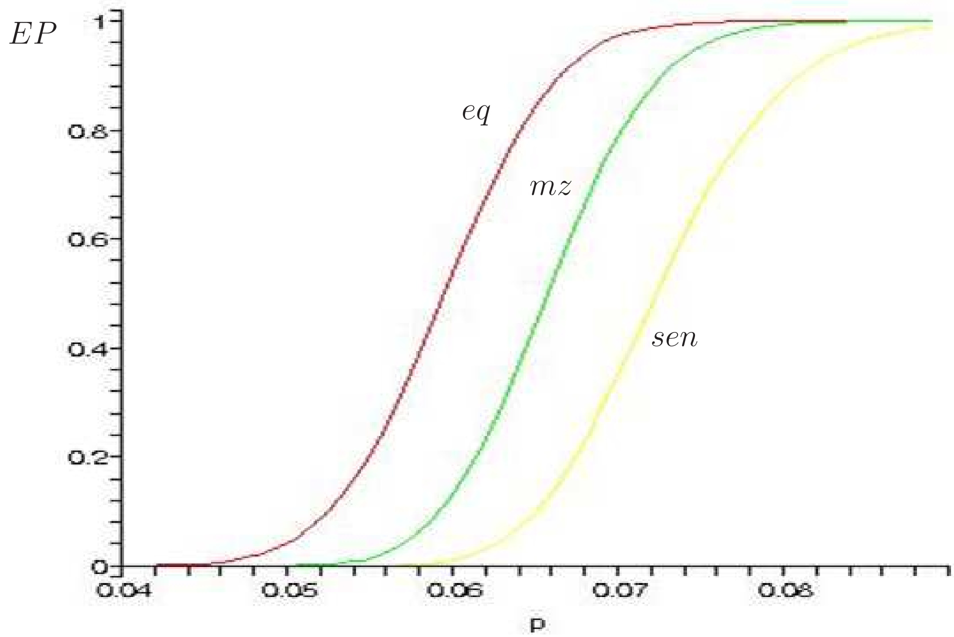

Figure 4: Exeedance Probabilities of the squared tranches

2 Literature

References

[1] Coval, C. D., Jurek, J. and Stafford, E. (2008) *The Economics of Structured Finance* Harvard Business School Finace Working Paper No 09-060,

[2] Whetten, M. und Adelson, M. (2005) *CDOs-Squared Demystified* Nomura Fixed Income Research