Statistics – B

Exercise 1

1) In class A, we have 17 students with the following heights [cm]

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Height	158	177	166	192	177	163	185	189	166	199	156	172	184	182	178	179	182

In class B, we have 12 students with the following age [years]

No.	1	2	3	4	5	6	7	8	9	10	11	12
Age	19	27	45	22	23	28	24	33	24	31	29	31

a) Calculate the median and the arithmetic mean of class A and B
b) Take class A and calculate range, MAD, unbiased and biased variance, uncorrected and corrected standard deviation.

2) In the last four years, a stock market investment had the following performance +10%, -15%, +12%, +7% p.a.. Within a savings contract during the last four years, an investment of 1200 Euro reached 1350 Euro in the end.

a) Compare the two investments via their yield p.a..

3) Consider two firms (A and B) issuing no new capital in the medium term with the following market capitalization P_i and net income E_i (i=A,B): P_A=200 Bil. Euro and E_A=8 Bil. Euro; P_B=800 Mil. Euro and and E_B=4 Mil. Euro. In this case P_i/E_i can be interpreted as the price earnings ratio of a firm.

a) Consider an Index consisting of 50% of firm A and 50% of firm B. Calculate P/E of the Index.

4) Show the following three properties of the arithmetic mean:

a) $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$
b) $x_i' = ax_i + b \rightarrow \bar{x}_i' = a\bar{x} + b$
c) $\sum_{i=1}^{n}(x_i - \bar{x})^2 \leq \sum_{i=1}^{n}(x_i - m)^2$ (for all m)

5) Show that $Var(\bar{x})$ is scaling with $\frac{1}{n}$ with respect to $Var(x_i) = \sigma^2$ resulting in $Var(\bar{x}) = \frac{\sigma^2}{n}$.

6) Show that $\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator ($E(\hat{\sigma}^2) = \sigma$) for the true Variance σ of a distribution if the expected value μ is unknown.

7) Take the following data set:

X	0	1	2	3
Number	10	20	50	20

a) Calculate median, arithmetic mean, unbiased variance, unbiased standard deviation, skewness and kurtosis

8) Take the weekly yields of the DAX since 1999 (data, i.e. googlefinance: =GOOGLEFINANCE("INDEXDB:DAX";"price";"01.01.1999";"XX.XX.2024";"weekly")) and calculate the skewness and kurtosis. Interprete from a descriptive point of view the distribution.

9) Take the following distributions and calculate the kurtosis. Interprete!

X	Counts	Counts	Counts	Counts	Counts
5	30	25	25	5	1
10	0	15	25	25	25
15	30	25	25	5	1